STEADY NON-ISOTHERMAL TWO-DIMENSIONAL FLOW OF NEWTONIAN FLUID IN A STENOISED CHANNEL

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ABSTRACT: *In present investigation the steady two-dimensional flow of an incompressible Newtonian fluid between two parallel plates with heat transfer in the presence of stenosis of cosine shape is studied. The governing equations are transformed into compatibility and energy equations, which are solved analytically with the help of Adomian Decomposition Method (ADM) and Regular Perturbation Method (RPM). The solutions obtained from the present analysis are given in terms of wall shear stress, separation point and temperature distribution through stenoised channel. The accuracy of the results is verified through available literature. It is found that wall shear stress and temperature increases with the development of stenosis and causing separation and reattachment in the region. It is observed that even at low velocity, separation occurs if the thickness of the stenosis is increased. Detailed discussion and graphical representations are also provided.*

Keywords:: Newtonian fluid, ADM, RPM, stenoised channel, heat transfer.

1. INTRODUCTION

The motivation of this study comes from the investigation of abnormal blood flow in the stenoised artery, which may be due to atherosclerotic plaques developed at various locations in the artery. Its effect on flow of blood is discussed by many authors theoretically, experimentally as well as numerically. Forrester and Young [1] presented the theoretical as well as experimental results of the axisymmetric, steady flow through converging and diverging tube with mild stenosis. Morgan and Young [2] provided the approximate analytical solution of axisymmetric, steady flow of incompressible Newtonian fluid both for mild and severe stenosis by using an integral method; basically they presented an extension of Forrester. Analysis of blood flow using incompressible Newtonian fluid through an axisymmetric stenoised artery of cosine shape is given by Haldar [3]. Layek and Midya [4] presented the numerical solution of time dependent incompressible Newtonian fluid for symmetric stenosis in two dimensional channels. Chow and Soda [5] analyzed the steady laminar flow of incompressible Newtonian fluid for different physical quantities by considering the sinusoidal boundary. The abnormal flow conditions developed due to stenosis can be an important factor in the development and progression of arterial diseases. Some of further major complications developed through this stenosis are the growth of tissues into arteries, development of an intravascular clot and post-stenotic dilatation. This type of flow also has applications in various fields like physiological flows and polymer science.

In present paper, the effect of stenosis height and Reynolds number on flow characteristics, wall shear stress, separation and reattachment points and heat transfer are analyzed. The study of Peclet number and Brinkman number on temperature distribution are also presented. It is observed that the general patron of flow is similar to the results given in [3 - 5]. The results of present investigation indicate that even a mild collar like stenosis in a small artery can create significant abnormalities in the flow including the phenomenon of separation.

This study presents the steady two-dimensional motion of incompressible Newtonian fluid in a cosine shape stenoised channel with heat transfer. In this analysis, we make a comparative study for the performance of ADM and RPM for highly non-linear compatibility equation in the geometry of a stenoised channel. The layout of the paper is as follows: The basic equations governing the flow, in the Cartesian coordinate, are given in section 2. Problem formulation is presented in Section 3. Section 4 and onward are dedicated for solution of different parameters. Section 5 provides graphical discussion. Conclusion is given in section 6.

2. BASIC EQUATIONS

The basic equations governing steady two dimensional flow of non-isothermal, incompressible Newtonian fluid in the absence of body forces are

$$\tilde{\vec{\nabla}} \cdot \tilde{\vec{V}} = 0, \qquad (1)$$

$$\rho \frac{d\tilde{\vec{V}}}{dt} = -\tilde{\vec{\nabla}}\tilde{p} + \tilde{\vec{\nabla}} \cdot \tilde{\tau}, \qquad (2)$$

$$\rho c_p \frac{d\tilde{T}}{dt} = k\tilde{\nabla}^2 \tilde{T} + \phi, \qquad (3)$$

where $\tilde{\vec{V}}$, $\tilde{T}$ and ρ are the velocity vector, temperature and constant density of fluid respectively, $\tilde{p}$ the dynamic pressure, c_p and k are specific heat and thermal conductivity parameters, $\tilde{\nabla}^2$ the Laplacian, ϕ the viscous dissipation function defined as $\phi = \tilde{\tau} \cdot \tilde{\nabla}\tilde{V}$ and $\frac{d}{dt}$ the material time derivative given as

$$\frac{d(\cdot)}{dt} = \frac{\partial(\cdot)}{\partial \tilde{t}} + \tilde{u}\frac{\partial(\cdot)}{\partial \tilde{x}} + \tilde{v}\frac{\partial(\cdot)}{\partial \tilde{y}}, \qquad (4)$$

where $\tilde{u}$ and $\tilde{v}$ are the velocity components in $\tilde{x}$ and $\tilde{y}$ directions and $\tilde{\tau}$ is extra stress tensor defined as

$$\tilde{\tau} = \mu \tilde{A}_1, \qquad (5)$$

where μ is dynamic viscosity and $\tilde{A}_1$ is first Rivlin-Ericksen tensor defined as

$$\tilde{A}_1 = \tilde{\vec{\nabla}}\tilde{V} + (\tilde{\vec{\nabla}}\tilde{V})^*,$$

where superscript * is defined for the transpose of tensor.

3. PROBLEM FORMULATION

Consider the non-isothermal Newtonian fluid flow through the channel of infinite length having stenosis of length $l_o/2$. The coordinate system is chosen in such a way that the arterial system lies in $\tilde{x}\tilde{y}-\text{plane}$, such that $\tilde{x}-\text{axis}$ coincide with center line in the direction of flow and $\tilde{y}-\text{axis}$ perpendicular to $\tilde{x}-\text{axis}$. Consider the boundary of stenoised region of the form [3] as follows

$$h(\tilde{x}) = \begin{cases} h_o - \dfrac{\lambda}{2}\left(1+\cos\left(\dfrac{4\pi\tilde{x}}{l_o}\right)\right) & -\dfrac{l_o}{4} < \tilde{x} < \dfrac{l_o}{4}, \\ h_o & \text{otherwise}, \end{cases} \tag{6}$$

.

where $h(\tilde{x})$ is variable gap due to stenosis, $2h_o$ the width of an unobstructed channel and λ the maximum height of stenosis. Boundary conditions for present problem are

$$\tilde{u} = \tilde{v} = 0,\ \tilde{T} = T_1 \quad at \quad \tilde{y} = h(\tilde{x}),$$
$$\frac{\partial \tilde{u}}{\partial \tilde{y}} = 0,\ \frac{\partial \tilde{T}}{\partial \tilde{y}} = 0 \quad at \quad \tilde{y} = 0,$$
$$\tilde{Q} = \int_0^{h(\tilde{x})} \tilde{u}\, d\tilde{y} = -\frac{1}{2}u_o h_o, \tag{7}$$

where u_o is the average velocity and $\tilde{Q}$ the volume flow rate

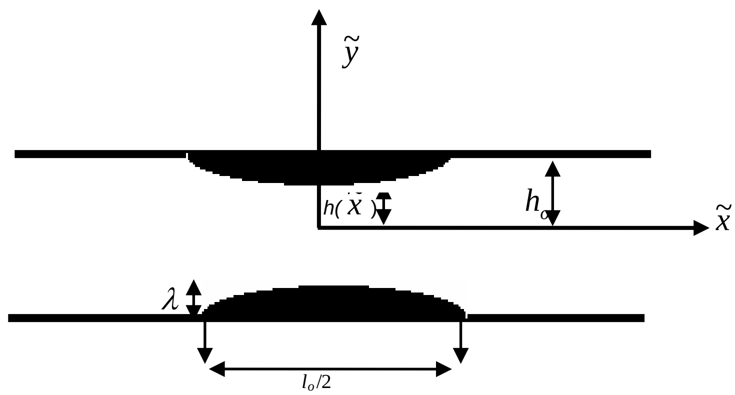


Figure 1: Geometry of the problem.

Velocity profile for steady homogeneous, two dimensional flows is assumed of the form

$$\tilde{\vec{V}} = \left(\tilde{u}(\tilde{x},\tilde{y}),\ \tilde{v}(\tilde{x},\tilde{y}),\ 0\right). \tag{8}$$

Introducing the dimensionless quantities as follows

$$x = \frac{\tilde{x}}{l_o},\ y = \frac{\tilde{y}}{h_o},\ u = \frac{\tilde{u}}{u_o},\ v = \frac{\tilde{v}}{u_o},\ p = \frac{h_o^2}{\mu u_o l_o}\tilde{p},\ \theta = \frac{\tilde{T}-T_o}{T_1 - T_o}, \tag{9}$$

where T_1 and T_o are temperatures on the boundary of stenosis and fluid respectively.

Substituting equations (4) - (5) in equations (1) – (3) and making use of (8)-(9), nondimensional form of basic equations are obtained as follows

$$\delta\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{10}$$

$$\text{Re}\left(\delta u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \nabla^2 u, \tag{11}$$

$$\text{Re}\,\delta\left(\delta u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \delta\nabla^2 v, \tag{12}$$

$$\text{Pe}\left(\delta u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right) = \nabla^2\theta + \text{Br}\left(4\delta^2\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \delta\frac{\partial v}{\partial x}\right)^2\right), \tag{13}$$

where

$$\delta = \frac{h_o}{l_o},\ \text{Re} = \frac{u_o h_o}{\nu},\ \text{Br} = \frac{u_o^2\mu}{k(T_1 - T_o)},\ \text{Pe} = \frac{\rho u_o h_o c_p}{k},$$

in which Re is the Reynolds number, Br the Brinkman number, Pe the Peclet number.

Now introducing the stream function as

$$u = \frac{\partial\psi}{\partial y},\ v = -\delta\frac{\partial\psi}{\partial x}, \tag{14}$$

which satisfy the continuity equation (10) identically. After eliminating pressure gradient term from momentum equations (11) - (12) and compatibility equation is obtained as

$$\text{Re}\,\delta\frac{\partial(\psi,\nabla^2\psi)}{\partial(y,x)} = \nabla^4\psi, \tag{15}$$

and energy equation takes the form

$$\nabla^2\theta = \mathrm{Pe}\delta\frac{\partial(\psi,\theta)}{\partial(y,x)} - \mathrm{Br}\left(4\delta^2\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2 + \left(\frac{\partial^2\psi}{\partial y^2} - \delta^2\frac{\partial^2\psi}{\partial x^2}\right)^2\right) - \delta^2\frac{\partial^2\theta}{\partial x^2} \quad (16)$$

where $\nabla^2 = \delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, is dimensionless form of Laplacian. The dimensionless form of (6) is

$$f = 1 - \frac{\varepsilon}{2}\left(1 + \cos(4\pi x)\right) \qquad -\frac{1}{4} < x < \frac{1}{4},$$
$$= 1 \qquad \text{otherwise}, \quad (17)$$

where $f = \frac{h(\tilde{x})}{h_o}$ and $\varepsilon = \lambda / h_o$.

Boundary conditions (7) in terms of stream function becomes

$$\frac{\partial\psi}{\partial y} = 0, \quad \psi = -\frac{1}{2}, \quad \theta = 1 \qquad at \qquad y = f,$$
$$\frac{\partial^2\psi}{\partial y^2} = 0, \quad \psi = 0, \quad \frac{\partial\theta}{\partial y} = 0 \qquad at \qquad y = 0. \quad (18)$$

Due to non-linearity of (15) and (16), it is difficult to find the exact solution so ADM and RPM are applied to find the analytical solution along with boundary conditions defined in equation (18).

4. SOLUTION

To solve the compatibility equation (15) and energy equation (16) along with boundary conditions (18), two methods namely, ADM and RPM are applied to find the series solutions. ADM does not require the small parameter, Cherruault [6] addressed extensively the concept of rapid convergence of ADM and Bellomo and Monaco [7], gives a useful comparison between ADM and Perturbation method and showed the efficiency of ADM compared to the tedious work required by Perturbation method. Convergence of ADM are shown by Adomian [8]-[9]. In RPM the flow variables ψ and θ are perturbed by taking δ as a small parameter.

4.1 Solution of compatibility equation by ADM:

The compatibility equation is

$$L_1\psi = \mathrm{Re}\,\delta\frac{\partial(\psi,\nabla^2\psi)}{\partial(y,x)} - \delta^4\frac{\partial^4\psi}{\partial x^4} - 2\delta^2\frac{\partial^4\psi}{\partial x^2\partial y^2}, \quad (19)$$

where L_1 is Adomian operator defined as

$$L_1 = \frac{\partial^4}{\partial y^4}, \quad (20)$$

and the inverse operator is

$$L_1^{-1}(*) = \iiint\int(*)dydydydy. \quad (21)$$

Apply L_1^{-1} to equation (19), we get

$$\psi = \frac{A}{6}y^3 + \frac{B}{2}y^2 + Cy + D + L_1^{-1}\left[\mathrm{Re}\,\delta\frac{\partial(\psi,\nabla^2\psi)}{\partial(y,x)} - \delta^4\frac{\partial^4\psi}{\partial x^4} - 2\delta^2\frac{\partial^4\psi}{\partial x^2\partial y^2}\right], \quad (22)$$

where A, B, C, D are functions of x to be determined.
Now to find the solution for different order, substituting

$$\psi = \sum_{n=0}^{\infty}\psi_n(x,y), \quad (23)$$

in equation (22), we obtain

$$\sum_{n=0}^{\infty}\psi_n(x,y) = \frac{A}{6}y^3 + \frac{B}{2}y^2 + Cy + D + L_1^{-1}\left[\mathrm{Re}\,\delta\frac{\partial\left((\sum_{n=0}^{\infty}\psi_n(x,y)),\nabla^2(\sum_{n=0}^{\infty}\psi_n(x,y))\right)}{\partial(y,x)} - \delta^4\frac{\partial^4(\sum_{n=0}^{\infty}\psi_n(x,y))}{\partial x^4} - 2\delta^2\frac{\partial^4(\sum_{n=0}^{\infty}\psi_n(x,y))}{\partial x^2\partial y^2}\right], \quad (24)$$

and boundary conditions becomes

$$\frac{\partial\left(\sum_{n=0}^{\infty}\psi_n(x,y)\right)}{\partial y} = 0, \quad \sum_{n=0}^{\infty}\psi_n(x,y) = -\frac{1}{2}, \quad \theta = 1 \qquad at \qquad y = f,$$
$$\frac{\partial^2\left(\sum_{n=0}^{\infty}\psi_n(x,y)\right)}{\partial y^2} = 0, \quad \sum_{n=0}^{\infty}\psi_n(x,y) = 0, \quad \frac{\partial\theta}{\partial y} = 0 \qquad at \qquad y = 0. \quad (25)$$

From (24), the zeroth order solution is

$$\psi_o = \frac{A}{6}y^3 + \frac{B}{2}y^2 + Cy + D, \quad (26)$$

and the boundary conditions for ψ_o becomes

$$\frac{\partial\psi_o}{\partial y} = 0, \quad \psi_o = -\frac{1}{2}, \qquad at \qquad y = f,$$
$$\frac{\partial^2\psi_o}{\partial y^2} = 0, \quad \psi_o = 0, \qquad at \qquad y = 0. \quad (27)$$

By using these boundary conditions, we obtain the solution for ψ_o as

$$\psi_o = \frac{\eta}{4}\left(\eta^2 - 3\right), \quad \text{where} \quad \eta = \frac{y}{f} \tag{28}$$

Now the recursive relation form (24) is

$$\psi_{n+1}(x,y) = L_1^{-1}\left[\begin{array}{l} \operatorname{Re}\delta \dfrac{\partial\left(\psi_n(x,y), \nabla^2\psi_n(x,y)\right)}{\partial\left(y,x\right)} - \\ \delta^4 \dfrac{\partial^4 \psi_n(x,y)}{\partial x^4} - 2\delta^2 \dfrac{\partial^4 \psi_n(x,y)}{\partial x^2 \partial y^2} \end{array}\right], \tag{29}$$

the expressions for ψ_1 and ψ_2 from (29) are

$$\psi_1(x,y) = L_1^{-1}\left[\begin{array}{l} \operatorname{Re}\delta \dfrac{\partial\left(\psi_o(x,y), \nabla^2\psi_o(x,y)\right)}{\partial\left(y,x\right)} - \\ \delta^4 \dfrac{\partial^4 \psi_o(x,y)}{\partial x^4} - 2\delta^2 \dfrac{\partial^4 \psi_o(x,y)}{\partial x^2 \partial y^2} \end{array}\right], \tag{30}$$

and

$$\psi_2(x,y) = L_1^{-1}\left[\begin{array}{l} \operatorname{Re}\delta \left\{\begin{array}{l} \dfrac{\partial\left(\psi_o(x,y), \nabla^2\psi_1(x,y)\right)}{\partial\left(y,x\right)} + \\ \dfrac{\partial\left(\psi_1(x,y), \nabla^2\psi_o(x,y)\right)}{\partial\left(y,x\right)} \end{array}\right\} - \\ \delta^4 \dfrac{\partial^4 \psi_1(x,y)}{\partial x^4} - 2\delta^2 \dfrac{\partial^4 \psi_1(x,y)}{\partial x^2 \partial y^2} \end{array}\right]. \tag{31}$$

The solution of ψ_1 is obtained by substituting ψ_o and integrating four times with respect to y of the form

$$\psi_1 = \frac{\delta\eta}{26880}\left[\begin{array}{l} \left(\eta^2 - 1\right)^2 \left\{\begin{array}{l} 288\delta^3 f f'^2 f''\left(10\eta^2 - 1\right) - 8\delta^2 f'^3\left(\operatorname{Re}\left(5\eta^4 - 17\eta^2 + 24\right) + 72 f'^4\left(5\eta^2 + 3\right)\right) \\ + 3f'\left(3\operatorname{Re}\delta^2 f f''\left(5\eta^4 - 18\eta^2 + 29\right) - 24\operatorname{Re}\left(\eta^2 - 5\right) - 64\delta^2 f^2 f'''\left(2\eta^2 - 3\right)\right) \\ - 144\delta^3 f^2 f''^2\left(2\eta^2 - 3\right) - \operatorname{Re}\delta f^2 f'''\left(5\eta^4 - 26\eta^2 + 69\right) \end{array}\right\} \\ + 6\delta f f''\left(6\eta^7 - 28\eta^5 + 49\eta^2 - 27\right) - 48\delta f'^2\left(3\eta^7 - 7\eta^5 + 7\eta^2 - 3\right) + 24\delta^2 f^2 f^{(4)}\left(\eta^2 - 5\right) \end{array}\right]. \tag{32}$$

Similarly we can find the solution for ψ_2 by substituting ψ_1 and ψ_o in equation (31) and integrating w.r.t. y. Equation (23) gives the stream function and (14) gives the velocity components u and v.

4.2 Solution of Energy Equation by ADM

Dimensionless form of energy equation in terms of stream function is

$$L_2\theta = \operatorname{Pe}\delta \frac{\partial(\psi,\theta)}{\partial(y,x)} - \operatorname{Br}\left(4\delta^2\left(\frac{\partial^2\psi}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2\psi}{\partial y^2} - \delta^2\frac{\partial^2\psi}{\partial x^2}\right)^2\right) - \delta^2\frac{\partial^2\theta}{\partial x^2}, \tag{33}$$

where

$$L_2 = \frac{\partial^2}{\partial y^2}, \tag{34}$$

is an Adomian operator and inverse operator is defined as

$$L_2^{-1}(*) = \int\int (*)dydy. \tag{35}$$

Apply L_2^{-1} to equation (33), we get

$$\theta = c_1 y + c_2 + L_2^{-1}\left\{\begin{array}{l} \operatorname{Pe}\delta \dfrac{\partial(\psi,\theta)}{\partial(y,x)} - \operatorname{Br} \\ \left(4\delta^2\left(\dfrac{\partial^2\psi}{\partial x\partial y}\right)^2 + \left(\dfrac{\partial^2\psi}{\partial y^2} - \delta^2\dfrac{\partial^2\psi}{\partial x^2}\right)^2\right) - \delta^2\dfrac{\partial^2\theta}{\partial x^2} \end{array}\right\}. \tag{36}$$

Substituting

$$\theta = \sum_{n=0}^{\infty}\theta_n(x,y), \; \psi = \sum_{n=0}^{\infty}\psi_n(x,y), \tag{37}$$

in equation (36) and comparing both sides, we obtain

$$\theta_o = c_1 y + c_2, \tag{38}$$

where c_1 and c_2 are functions of x to be determine and first and second order temperatures are

$$\theta_1 = L_2^{-1}\left\{ \begin{array}{l} \mathrm{Pe}\delta \dfrac{\partial(\psi_o,\theta_o)}{\partial(y,x)} - \mathrm{Br} \\ \left(\begin{array}{l} 4\delta^2\left(\dfrac{\partial^2\psi_o}{\partial x\partial y}\right)^2 \\ +\left(\dfrac{\partial^2\psi_o}{\partial y^2} - \delta^2\dfrac{\partial^2\psi_o}{\partial x^2}\right)^2 \end{array} \right) - \delta^2\dfrac{\partial^2\theta_o}{\partial x^2} \end{array} \right\}, \quad (39)$$

$$\theta_2 = L_2^{-1}\left\{ \begin{array}{l} \mathrm{Pe}\delta\left(\dfrac{\partial(\psi_o,\theta_1)}{\partial(y,x)} + \dfrac{\partial(\psi_1,\theta_o)}{\partial(y,x)}\right) - 2\mathrm{Br} \\ \left(\begin{array}{l} 4\delta^2\dfrac{\partial^2\psi_o}{\partial x\partial y}\dfrac{\partial^2\psi_1}{\partial x\partial y} + \dfrac{\partial^2\psi_o}{\partial y^2}\dfrac{\partial^2\psi_1}{\partial y^2} + \delta^4\dfrac{\partial^2\psi_o}{\partial x^2}\dfrac{\partial^2\psi_1}{\partial x^2} \\ -\delta^2\left(\dfrac{\partial^2\psi_o}{\partial y^2}\dfrac{\partial^2\psi_1}{\partial x^2} + \dfrac{\partial^2\psi_1}{\partial y^2}\dfrac{\partial^2\psi_o}{\partial x^2}\right) \end{array} \right) - \delta^2\dfrac{\partial^2\theta_1}{\partial x^2} \end{array} \right\}, \quad (40)$$

subject to the boundary conditions on temperature are

$$\sum_{n=0}^{\infty}\theta_n(x,y) = 1 \quad at \quad y = f$$

$$and \quad \frac{\partial\left(\sum_{n=0}^{\infty}\theta_n(x,y)\right)}{\partial y} = 0 \quad at \quad y = 0. \quad (41)$$

Equation (38) and (39) gives solution as

$$\theta_o = 1, \quad (42)$$

$$\theta_1 = -\frac{3Br(\eta^2-1)}{4480f^2}\left\{ \begin{array}{l} 280(\eta^2+1) + 8\delta^4 f'^4(30\eta^6 - 26\eta^4 + 9\eta^2 + 9) + 56\delta^2 ff''(2\eta^4 - 3\eta^2 - 3) \\ +\delta^4 f^2 f''^2(15\eta^6 - 41\eta^4 + 29\eta^2 + 29) + 8\delta^4 f'^2\left(\begin{array}{l} 70(\eta^4 - \eta^2 + 2) \\ -\delta^2 ff''(15\eta^6 - 27\eta^4 + 8\eta^2 + 8) \end{array} \right) \end{array} \right\}, \quad (43)$$

which depends upon ratio of heat production by viscous dissipation to heat transport by conduction. Similarly, we can find the expression for θ_2 from equation (40).

Dimensionless form of wall shear stress

$$T_w = \left(\frac{\partial u}{\partial y} + \delta\frac{\partial v}{\partial x}\right)_{y=f} \quad (44)$$

and solution up to second order is

$$T_w = \frac{1}{f^2}\left[\begin{array}{l} \dfrac{3}{2} + \dfrac{3\,\mathrm{Re}\, f'}{35}\delta + \delta^2 \\ \left\{ \dfrac{\mathrm{Re}^2}{26950}\left(40ff'' - 79f'^2\right) + \dfrac{3}{10}\left(2ff'' - 13f'^2\right) \right\} \end{array} \right]. \quad (45)$$

The points of separation and reattachment are the points at which the back flow occurs or the wall shear stress is zero, i.e. $T_\omega = 0$, then equation (45) reduces in terms of Reynolds number as

$$\mathrm{Re} = \frac{1}{\delta\left(40ff'' - 79f'^2\right)}\left\{-1155f' \pm \sqrt{1334025f'^2 - \left(40ff'' - 79f'^2\right)\left(40425 - 105105\delta^2 f'^2 + 16170\delta^2 ff''\right)}\right\} \quad (46).$$

By using equation (46), our aim is to find graphically the critical Reynolds number at which the back flow occurs.

5. GRAPHICAL DISCUSSION

In this section the effect of different pertinent parameters on wall shear stress, separation and reattachment points and analysis for heat transfer are presented graphically. The effect of Re on wall shear stress in the channel is shown in figures 3a, 3b. It is observed that the wall shear stress becomes high on the stenoised region with increasing Re. The negative shearing corresponds to back flow which causes separation and reattachment points in the channel. It is observed that both the methods ADM and RPM give the same graphical results.

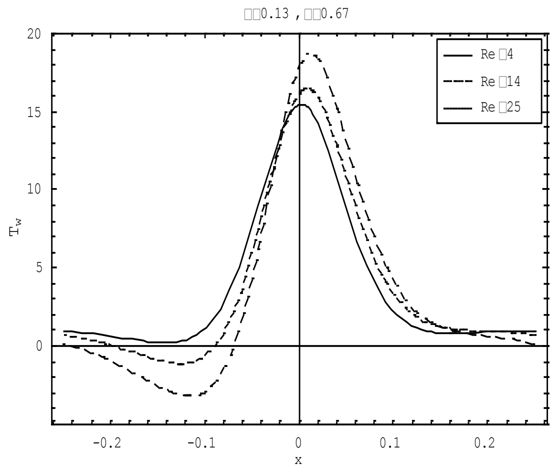
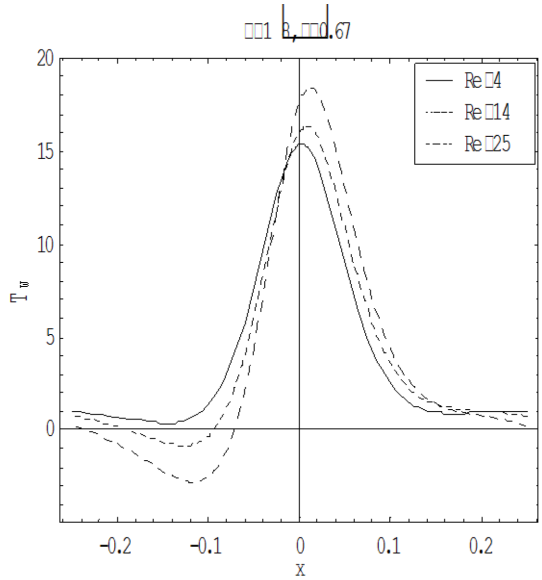


Figure 3a: Effect of Re on wall shear stress (ADM)



. **Figure 3b: Effect of Re on wall shear stress (RPM).**

The effect of stenosis thickness ε on wall shear stress is presented in figure 4a, 4b. It is observed that with the increase in ε increases wall shear stress in the stenoised region and also responsible for back flow. The straight line presents the flow for $\varepsilon = 0$, which is known as fully developed or Poiseuille flow. Further more negative shearing is observed by ADM as compared to RPM.

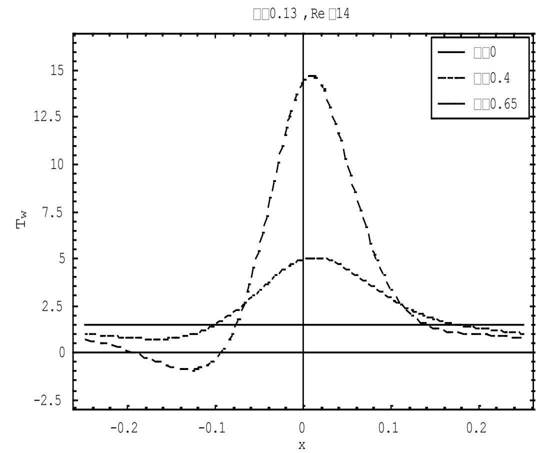


Figure 4a: Effect of ε on wall shear stress (ADM).

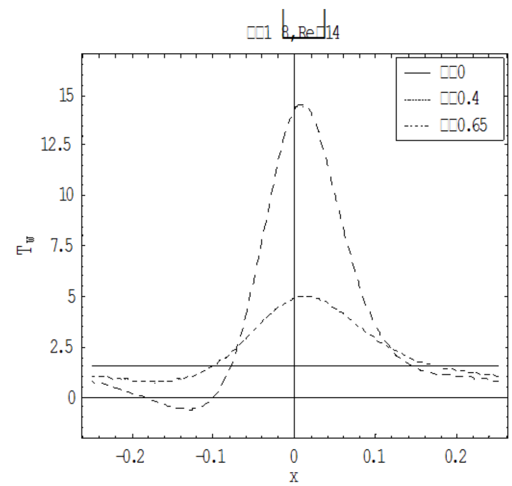


Figure 4b: Effect of ε on wall shear stress (RPM).

In Figures 5a, 5b zero wall shear stress is plotted for different values of ε in the converging and diverging sections of the channel. The aim is to find the critical value of Re, where separation and reattachment could be observed. As the critical Re reached the separation occurs in the converging region of channel and reattachment point in the downstream region of channel. It is observed from both the figures that with increasing ε the critical value of Re decreases.

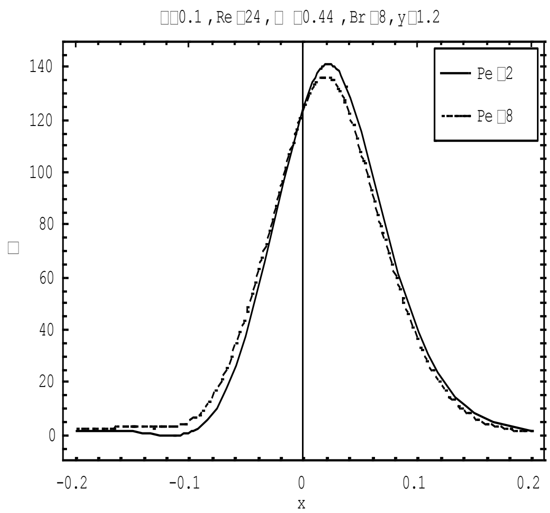


Figure 5a: Separation points in the converging region (ADM).

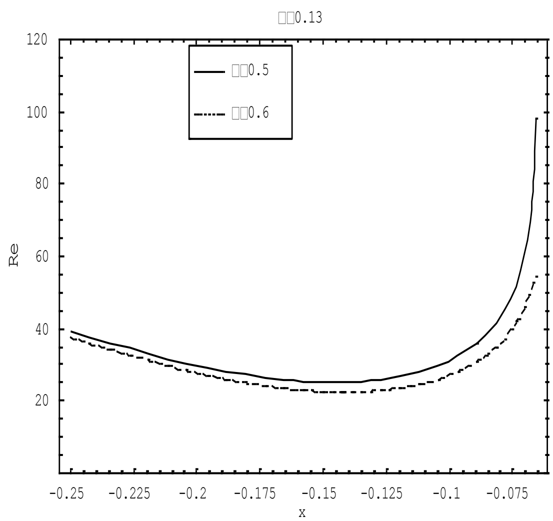


Figure 6a: Effect of Pe **on temperature dist. by ADM.**

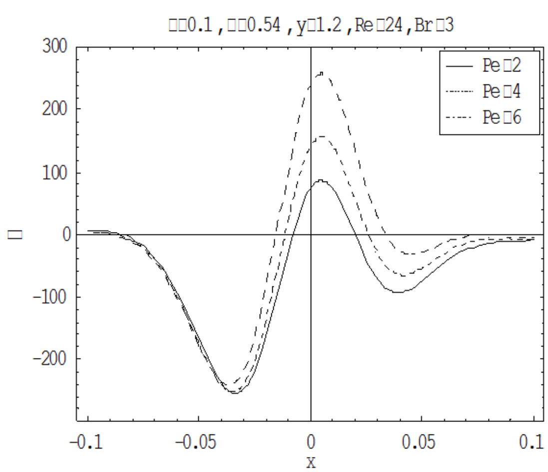


Figure 6b: Effect of Pe **on temperature dist. by RPM**

Figure 6a presents the effect of temperature distribution for Pe number by ADM. It is observed that with the increase in

Figure 5b: Separation points in the converging region (RPM).

Pe number temperature increases in the converging region and decreases in the diverging region. Figure 6b shows the effect of Pe in the converging and diverging sections on temperature distribution. It is found that with the increase in Pe temperature increases over the stenosis and becomes negative in the converging and diverging sections due to back flow. It is observed that ADM shows the better results as compared with RPM.

In figures 7a, 7b effects of Br on temperature distribution is presented. It is found that the temperature increases over the stenosis by the increase in Br along with the fixed values of the other parameters and becomes negative due to reverse flow. It is observed that ADM gives the better results than RPM.

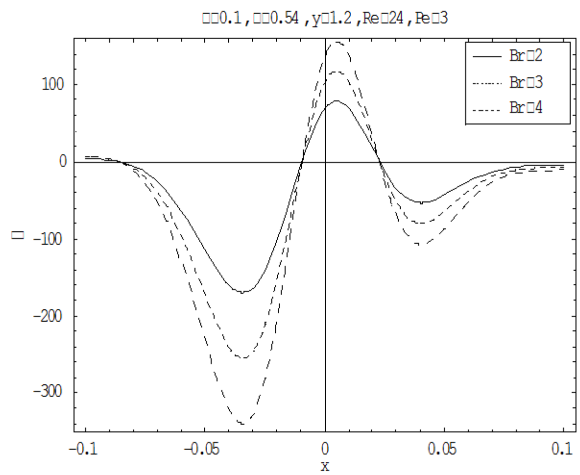


Figure 7a: Effect of Br **on temperature dist. by ADM. Figure**

7b: Effect of Br **on temperature dist.(RPM)**

6. CONCLUSION

In the present study, steady two-dimensional flow of an incompressible Newtonian fluid between two parallel plates with heat transfer in the presence of stenosis of cosine shape is presented. Underlying problem is solved with the help of ADM and RPM. The results thus obtained are discussed graphically in terms of wall shear stress, separation and reattachment point, temperature distribution. It is observed that the wall shear stress is same as given by [2 -3] and separation and reattachment points are in agreement with [3]. It is also observed that:

(i) Increase in Reynolds number increases the wall shear stress.

(ii) Increase in thickness of stenosis increases wall shear stress causing separation and reattachment in the channel.

(iii) Development in the thickness of stenosis decreases the critical Reynolds number for separation and reattachment points, means even at low velocity, separation is observed if the thickness of stenosis increases.

(iv) By the increase in Peclet and Brinkman number increases the temperature in the channels.

(v) For $\varepsilon = 0$, Poiseuille flow is recovered.

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